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Study of Application of De-Morgan's law in Modern Fields

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Abstract

This paper discusses about the De-Morgan's Law, with both the conditions, the law relates the intersection and union of sets through complements. In propositional logic De-Morgan's Laws relate conjunctions and disjunctions of propositions through negation. Use of the law in Boolean algebra has been demonstrated through logic diagram and application of the law in other fields have been described, the paper also discusses about the law along with some examples to prove the law.

Keywords

Boolean algebra, logic diagram, conditions, applications, proofs.

1. Introduction

De-Morgan's law is named after Augustus De Morgan, a 19th century British mathematician, who introduced a formal version of the laws to classical propositional logic. De-Morgan's Law is used both in set theory as well as in Boolean algebra. The objective of the current study is to verify the working of De-Morgan's Laws and explore the scope of the law in various dimensions.

1.1 De-Morgan's Laws

Boolean algebra has postulates and identities. We can frequently utilize these regulations to decrease articulation into a more helpful structure. One of these regulations is the De-Morgan's Law. De-Morgan's law has two conditions or laws:

First Condition or First Law

It states that the compliment of the product of two variables is equal to sum of the compliment of each variable.

$$(AB)' = A' + B'$$

Second Condition or Second Law

It states that the compliment of the sum of two variables is equal to the product of the compliment of each variable.

$$(A+B)' = A' \,.\, B'$$

2. Logic Diagram

The Logic Diagram used for the First Law of De-Morgan's law are as follows:

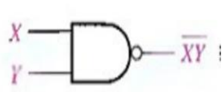

Figure 1. NAND Gate

$\equiv$

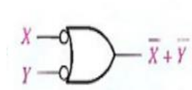

Figure 2. Negative-OR Gate

Table 1. Truth Table of first law

Input		Output	
X	**Y**	$(XY)'$	$(X+Y)'$
0	**0**	1	1
0	**1**	1	1
1	**0**	1	1
1	**1**	0	0

The Logic Diagram used for the Second Law of De-Morgan's law are as follows:

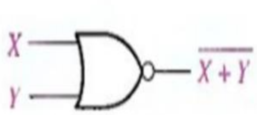

Figure 3. NOR GATE

$\equiv$

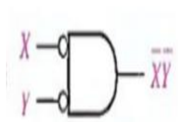

Figure 4. Negative-AND

Table 2. Truth Table of Second Law

Input		Output	
X	**Y**	$\overline{(X+Y)'}$	$(XY)'$
0	**0**	1	1
0	**1**	0	0
1	**0**	0	0
1	**1**	0	0

2.1. Equivalent gates of De-Morgan's Theorem

Based on De-Morgan's first and second theorems, it is observed that all the AND operators in a logical expression can be replaced with OR operators and vice versa and then inverts every term in the expression which means logic '0' to logic '1' and logic '1' to logic '0'. So, to get De-Morgan's equivalent gates for AND, OR, and NOT gates, inverters are to be added for all the inputs and outputs and alter AND to OR gate and OR to AND gate. The equivalent gates of De-Morgan's law are shown below by comparing with fundamental logical gates as shown in figures 5-12.

Fundamental Logic Gates	De-Morgan's Logic Gates

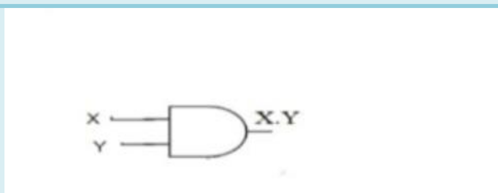

Figure 5. AND Gate

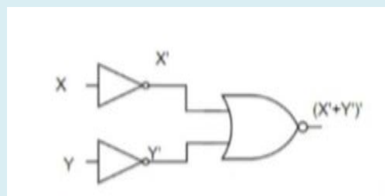

Figure 6. Negative NOR Gate

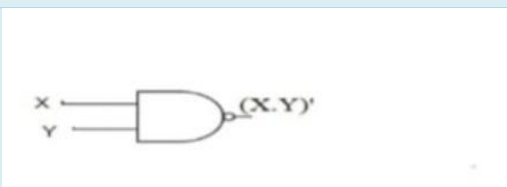

Figure 7. NAND Gate

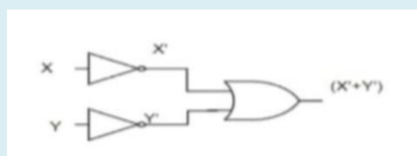

Figure 8. Negative OR Gate

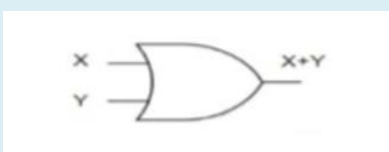

Figure 9. OR Gate

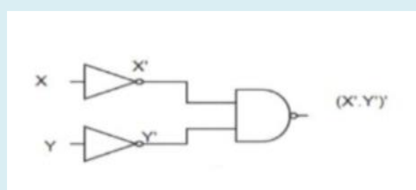

Figure 10. Negative NAND Gate

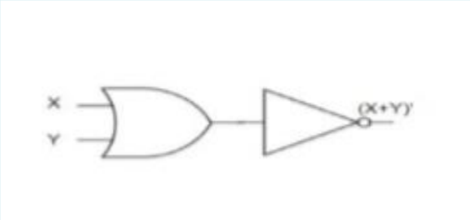

Figure 11. NOR Gate

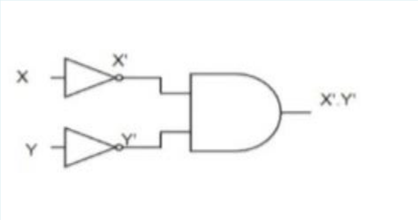

Figure 12. Negative AND Gate

3. Applications

The applications of De-Morgan's Theorem are crucially observed in multiple domains of engineering and mathematics.

a. In the domain of engineering, using De-Morgan's laws, Boolean expressions can be built easily only through one gate which is usually NAND or NOR gates. This results in hardware design at a cheaper cost.
b. Used in the verification of SAS code.
c. Implemented in computer and electrical engineering domain.
d. De-Morgan's laws are also employed in java programming.
e. The De-Morgan’s theorem is mostly used in digital programming and for making digital circuit diagrams.

De-Morgan’s Law in Java Programming

!(A&&B) == !A || !B

! (raining && cold) ==! raining ||! cold

!(let>=’A’&&let<=’z’) == let< ‘A’ || let > ‘Z’

!(A||B) == !A && !B

! (Tuesday || JAVA) ==! Tuesday && !JAVA

!(ans == ‘y’ ||ans == ‘Y’) == ans != ‘y’ && ans != ‘y’

3.1 Use of De-Morgan's law in other fields.

De-Morgan's law in Statistics -In the domain of statistics, it also needs set theory. The statement from the De-Morgan’s theorem defines interactions between several set theory functions. The laws are

$$(X \cap Y)' = X' \cup Y'$$

And

$$(\boldsymbol{X} \cup \boldsymbol{Y})' = \boldsymbol{X}' \cap \boldsymbol{Y}'$$

De-Morgan's in physics-When two or more input variables are first AND and then negated giving a NAND gate, they are equivalent to the OR of the complement of the individual variables. Thus , the equivalent NAND function will be negative OR function, providing that

A.B = A+B

De-Morgan's Law

Proving $(\mathrm{A} \cup \mathrm{B})' = \mathrm{A}' \cap \mathrm{B}'$

Proof - LHS $(\mathrm{A} \cup \mathrm{B})'$

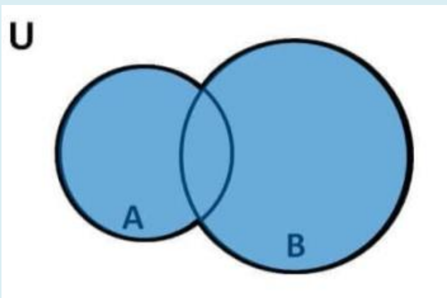

Figure 13. $\mathrm{A} \cup \mathrm{B}$

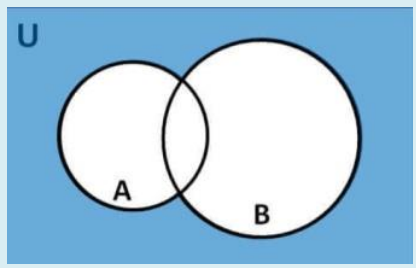

Figure 14. $(\mathrm{A} \cup \mathrm{B})'$

RHS $\mathrm{A}' \cap \mathrm{B}'$

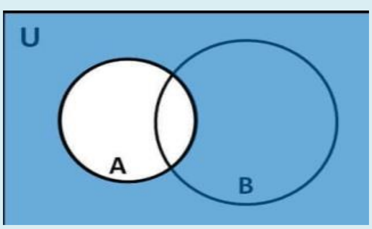

Figure 15. A'

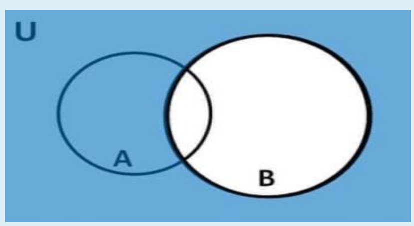

Figure 16. B'

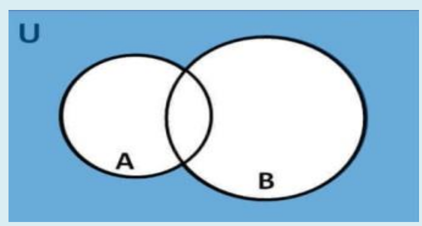

Figure 17. $\mathrm{A}' \cap \mathrm{B}'$

4. Examples

1) Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.

Show that (A ∪ B) ' = A ' ∩ B'.

Solution

U = {1, 2, 3, 4, 5, 6}

A = {2, 3}

B = {3, 4, 5}

A ∪ B = {2, 3} ∪ {3, 4, 5} = {2, 3, 4, 5}

∴ (A ∪ B) ' = {1, 6}

Also A ' = {1, 4, 5, 6}

B ' = {1, 2, 6}

∴ A' ∩ B' = {1, 4, 5, 6} ∩ {1, 2, 6}

= {1, 6}

Hence (A∪ B)' = A ' ∩ B'.

2) If ξ = {a,b,c,d,e}, A = { a,b,d} and B = {b,d,e}. Prove De Morgan's law of intersection.

Solution
ξ = {a,b,c,d,e}
A = { a,b,d}
B = {b,d,e}
(A ∩ B) = { a,b,d} ∩ {b,d,e}
(A ∩ B) = {b,d}
∴ (A ∩ B)' = {a, c,e} ----->(1)

A' = {c,e} and B' = {a,c}

∴ A' ∪ B' = {c,e} ∪ {a,c}
A' ∪ B'= { a, c,e} ----->(2)

From (1) and (2)

(A ∩ B)' = A' ∪ B' (which is a De Morgan's law of intersection.

5. Importance of De-Morgan's Law

- The theorems of De-Morgan's law have been proved to be very useful for simplifying Boolean logic expressions due to the way they can ‘break’ an inversion, which could be the complement of a complex Boolean expression.
- The theorems of De-Morgan's can also be used to express logic expressions that do not originally contain inversion terms differently. Also, this can again prove to be useful while simplifying the Boolean equations. When used in this way it must be taken care of to not to forget the final inversion, which can be easily avoided by complementing both the sides of the expression to be simplified before applying De-Morgan's theorem and then again complementing after simplification.
- Lastly, the students must note that one way of interpretating De-Morgan's theorem is that any AND/OR operation can be considered as an OR/AND operation as long as NOT gates are used as well in the equation for ease of calculations.
- De-Morgan's law states that the complement of the union of two sets is the intersection of their complement, and also the complement of intersection of two sets is the union of their complements.
- The main usage of De-Morgan's law is in designing digital circuits. These laws related to the union and intersection of sets using complements. It explains that the complement of the sum of all input variables equals the product of the complement of every term.
- The fundamental operations like the union of sets, the intersection of sets and the complement of sets are used in De-Morgan's law.
- De-Morgan's law is used for a better understanding of the multiple set operations and their inter-relationships in set theory.
- The De-Morgan's theorems are used for mathematical verification of the equivalency of the NOR and negative- AND gates and the negative-OR and NAND gates. These theorems play an important role in solving various Boolean algebra expressions.
- Complementation bars are proposed to operate as grouping symbols. Hence, when a bar is broken, then expression beneath it should remain grouped. Parentheses may be positioned around these grouped expressions as assistance to give a miss to changing precedence.

The main application of De-Morgan’s at the present time is the usage in designing digital circuits. The law can help to simplify the code to make it more readable. In game theory usage of this law is analyzes. De-Morgan’s law is used for better understanding of multiple set operations and their inter-relationship in set theory.

6. Conclusion

De-Morgan’s Law describes how mathematical statements and concepts are related through their opposites. De-Morgan’s law is used in both set theory and Boolean algebra. In set theory, De-Morgan’s law relates the intersection and union of sets through complements. It is used for better understanding of the multiple set operations and their inter-relationship in set theory. And in Boolean algebra, De-Morgan’s law explains that the complement of the product of all the terms is equal to the sum of the complement of each term. Thus, there are a lot of possibilities in the future to work on the above discussed ‘Applications of De-Morgan’s.

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