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M - Projective Curvature Tensor Equipped with a -kenmotsu Manifold

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Abstract

In this paper, we studied the properties of ϵ-Kenmotsu manifolds that possess an M -projective curvature tensor. We have shown that ϵ-Kenmotsu manifolds with an M -projectively flat and irrotational M -projective curvature tensor are locally isometric to the hyperbolic space Hn(c), where $c = -\epsilon^2$. Additionally, we have investigated the condition $R(X, Y) \cdot S = 0$ for M -projectively flat ϵ-Kenmotsu manifolds. Then we focused on the analysis of ϵ-Kenmotsu manifolds with a conservative M -projective curvature tensor. Lastly, we have certain geometric results for ϵ-Kenmotsu manifolds that satisfy the relation $M(X, Y) \cdot R = 0$.
2020 Mathematics Subject Classification *:53C05, 53C20, 53C25, 53D15,53D10.*

Keywords

Trans Sasakian manifold, ϵ- Kenmotsu manifold, M -projective curvature tensor, Einstein manifold, η- Einstein manifold , irrotational M - projective curvature tensor and con- servative M -projective curvature tensor

1. Introduction

The basic difference between Riemannian and semi-Riemannian geometry is the existence of a null vector. In a Riemannian manifold (M, g), the signature of the metric tensor is positive definite, whereas the signature of a semi-Riemannian manifold is indefinite. With the help of indefinite metric Bejancu and Duggal [1] introduced

ϵ-Sasakian manifolds. Then Xufeng and Xiaoli [13]proved that every ϵ-Sasakian manifold must be a real hyperface of some indefinite Kahler manifolds. Since Sasakian manifolds with indefinite metric have applications in Physics [1], we are interested to study various contact manifolds with indefinite metric. Geometry of Kenmotsu manifolds originated from Kenmotsu [10]. In [3] De and Sarkar introduced the notion of ϵ-Kenmotsu manifolds with indefinite metric. On the other hand, in [6] Eisenhart proved that if a Riemannian manifold admits a second order parallel syemmetric co-variant tensor other than a constant multiple of the metric tensor, then it is reducible. Later on, several authors investigated the Eisenhart problem on various spaces and obtained some interesting results. Recently, Haseeb and De [7] have studied η-Ricci solitons in ϵ-Kenmotsu manifolds. ϵ-Kenmotsu manifolds have also been studied by several authors such as ([2, 8, 9, 13, 15]) and many others. So far, our knowledge about curvature symmetries have not been studied in semi-Riemannian manifolds. In this paper, we are going to study curvature symmetries in ϵ-Kenmotsu manifolds. For curvature symmetries we refer the book of Duggal and Sharma [5]. Sharma [12] characterised a class of contact manifold admitting a vector field keeping the curvature tensor invariant.

Definition 1.1. *The M- projective curvature tensor of Riemannian manifold Mn was defined by Pokhariyal and Mishra [18] is of the following form:*

$$M(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY], \quad (1)$$

where Q is the Ricci operator defined on

$$S(X,Y) = g(QX,Y).$$

A space form (i.e., a complete simply connected Riemannian manifold of constant curvature) is said to be elliptic, hyperbolic or Euclidean according as the sectional curvature tensor is positive, nega- tive or zero [5]. The authors extensively studied the properties of M - projective curvature tensor on the various manifolds(see, [**?**, 9, 17, 20, 21, 26, 28]. In this paper, we have studied some special proper- ties of ϵ- Kenmotsu manifold.The purpose of this paper is to study the properties of M - projective curvature tensor in ϵ- Kenmotsu manifolds.

The paper is organized as follows: Section 2 is concerned with preliminaries of ϵ- Kenmotsu mani- folds. In section 3, we study the M -projectively flat of ϵ-Kenmotsu manifold. Section 4 deals with the M - projectively flat ϵ-Kenmotsu manifold satisfies the condition $R(X,Y) = 0$. In section 5, we study conservative M - projective curvature tensor of ϵ- Kenmotsu manifold. In section 6, irrota- tional M - projective curvature tensor of ϵ- Kenmotsu manifold are studied. Section 7 is devoted to studying - Kenmotsu manifold satisfies the condition $M(X,Y)R = 0$.

2. Preliminaries

An almost contact structure on a n-dimensional differentiable manifold M is a triple (ϕ, ξ, η), where ϕ is a tensor field of type (1, 1), η is a 1-form and ξ is a vector field such that

$$\phi^2 = -I + \eta\xi, \quad (2)$$

$$\eta(\xi) = 1, \phi\xi = 0, \eta\phi = 0. \quad (3)$$

A differentiable manifold with an almost contact structure is called an almost contact manifold. An almost contact metric manifold is an almost contact manifold endowed with a compatible metric g. An almost contact metric manifold M is said to be an ϵ-almost contact metric manifold if

$$g(\xi, \xi) = \pm 1 = \epsilon, \quad (4)$$

$$\eta(X) = \epsilon g(X, \xi), rank(\phi) = n - 1, \quad (5)$$

$$g(\phi X, \phi Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), X, Y \in (TM), \quad (6)$$

holds, where ξ is space-like or time-like but it is never a light like vector field. We say that (ϕ, ξ, η, g) is an ϵ-contact metric structure if we have

$$d\eta(X, Y) = g(X, \phi Y). \quad (7)$$

In this case, M is an ϵ-contact metric manifold. An ϵ-contact metric manifold is called an ϵ-Kenmotsu manifold [7] if

$$(\nabla_X\phi)Y = -g(X, \phi Y)\xi - \epsilon\eta(Y)\phi X, \quad (8)$$

holds, where ∇ is the Riemannian connection of g. An ϵ-almost contact metric manifold is a ϵ-Kenmotsu manifold if and only if

$$\nabla_X\xi = \epsilon(X - \eta(X)\xi). \quad (9)$$

The following conditions holds in an ϵ-Kenmotsu manifold [7]:

$$(\nabla_X\eta)(Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), \quad (10)$$

$$\eta(R(X, Y)Z) = \epsilon g(X, Z)Y - g(Y, Z)X, \quad (11)$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \quad (12)$$

$$R(\xi, X)Y = \eta(Y)X - \epsilon g(X, Y)\xi, \quad (13)$$

$$S(X, \xi) = -(n - 1)\eta(X), \quad (14)$$

$$Q\xi = -\epsilon(n - 1)\xi, \quad (15)$$

$$S(\phi X, \phi Y) = S(X, Y) + \epsilon(n - 1)\eta(X)\eta(Y). \quad (16)$$

for any vector fields X, Y, Z on M, where R, S and Q denotes the curvature tensor, Ricci tensor and Ricci operator on M.

Definition 2:. An ϵ- manifold M is said to be η-Einstein manifold if its Ricci tensor S is of the form

$$S(X, Y) = \lambda_1 g(X, Y) + \lambda_2\eta(X)\eta(Y), \quad (17)$$

for any vector fields X, Y, where λ_1, λ_2 are smooth functions on M.

If $\lambda_2 = 0$, then η-Einstein manifold becomes Einstein manifold. In view of (2) and (17), we have

$$QX = \lambda_1 X + \lambda_2\eta(X)\xi \quad (18)$$

Let us consider an ϵ-Kenmotsu manifold. Then putting $X = Y = e_i$ in (17), $i = 1, 2, ...n$ and taking summation for $1 \leq i \leq n$, we have

$$r = n\lambda_1 + \epsilon\lambda_2 \quad (19)$$

Now, setting $X = Y = \xi$ in (17) and using (2), (3) and (14), we obtain

$$-(n - 1) = \epsilon\lambda_1 + \lambda_2 \quad (20)$$

From the conditions (19) and (20), gives

$$\lambda_1 = \epsilon - \frac{r}{(1 - n)} \quad (21)$$

$$\lambda_2 = \frac{r}{\epsilon} - \frac{n\epsilon}{\epsilon} + \frac{nr}{(1 - n)\epsilon} \quad (22)$$

where, r is the scalar curvature.

In view of (8) − (11), it can be easily constructed that in n -dimensional ϵ-kenmotsu manifold M , the M -projective curvature tensor satisfies the following condition from (1.1):

$$M(X,Y)\xi = \frac{1}{2}[\eta(X)Y - \eta(Y)X] - \frac{1}{2(n-1)}[-(n-1)\eta(Y)X + (n-1)\eta(X)Y + \epsilon\eta(Y)QX - \epsilon\eta(X)QY]$$

$$M(\xi,X)Y = \frac{1}{2}[\eta(Y)X - \epsilon g(X,Y)\xi] - \frac{1}{2(n-1)}[S(X,Y)\xi - \epsilon\eta(Y)QX] \qquad (23)$$

3. M -Projectively flat ϵ-kenmotsu manifold

In this section, we study M-Projectively flat ϵ-kenmotsu manifold .

Definition 3.1. *The Lorentzian ϵ-kenmotsu manifold M is said to be a M-projectively flat, if we have*

$$M(X,Y)Z = 0. \qquad (24)$$

for any vector fields X, Y, Z on M.

By taking into account of relation (1) and using definition , we get

$$R(X,Y)Z = \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY], \qquad (25)$$

Taking $Z = \xi$ in (25) and using relations (3), (12) and (14),we have

$$\epsilon[\eta(X)Y - \eta(Y)X] = \frac{1}{n-1}[\eta(Y)QX - \eta(X)QY], \qquad (26)$$

Again putting $Y = \xi$ in (26) and using (2), (3) in (14), we get

$$QX = -(n-1)\epsilon X, \qquad (27)$$

which on simplification gives,

$$S(X,Y) = -(n-1)\epsilon g(X,Y), \qquad (28)$$

which yields,

$$r = -(n-1)\epsilon, \qquad (29)$$

Thus, we get the following theorem.

Theorem If an n-dimensional ϵ-kenmotsu manifold is M-Projectively flat ,then it is an Einstein manifold and its Ricci tensor of M has the form

$$S(X,Y) = -(n-1)\epsilon g(X,Y). \qquad (30)$$

In consequences of (28) ,(25) becomes

$$R(X,Y)Z = -\epsilon g(Y,Z)X - g(X,Z)Y. \qquad (31)$$

A Space form is said to be hyperbolic if the sectional curvature tensor is negative [5]. Thus, we can state

Theorem 3.1. *If an n-dimensional ϵ-kenmotsu manifold is M -Projectively flat ,then it is either locally isometric to the hyperbolic space $H(c)$,where $c = -\epsilon$ or M has the constant scalar curvature of the form $-(n-1)\epsilon$.*

4. M -Projectively ϵ-kenmotsu manifold satisfying the condi- tion R(X, Y).S = 0

In this section , we consider that manifold is an M -Projectively flat ϵ-kenmotsu manifold satisfying the condition $R(X, Y).S = 0$ .Thus we have

$$S(R(X, Y)Z, U) + S(Z, R(X, Y)U) = 0. \quad (32)$$

In view of (25) in (32), we have

$$\frac{1}{2(n-1)}[S(QX, U)g(Y, Z) - S(QY, U)g(X, Z) + S(QX, Z)g(Y, U) - S(QY, Z)g(X, U)] = 0. \quad (33)$$

Putting $Y = Z = \xi$ in (33) and using the relation (2), (3) and (14), then we have

$$\frac{1}{2(n-1)}[S(QX, U)g(\xi, \xi) - S(Q\xi, U)g(X, \xi) + S(QX, \xi)g(\xi, U) - g(X, U)S(Q\xi, \xi) = 0] \quad (34)$$

Again, using (14) in (34) ,we have

$$\epsilon S(QX, U) - (n-1)^2\eta(U)\eta(X) + \epsilon\eta(U)S(QX, \xi) - \epsilon(n-1)^2 g(X, U) = 0. \quad (35)$$

Let λ be the eigen value of endomorphism Q corresponding to an eigen-vector X. Then putting $QX = \lambda X$ in (35) and using the relation $g(QX, Y) = S(X, Y)$, then we find that

$$\epsilon\lambda^2 g(X, U) - (n-1)^2\eta(U)\eta(X) - \epsilon\lambda(n-1)\eta(U)\eta(X) - \epsilon(n-1)^2 g(X, U) = 0 \quad (36)$$

Now, putting $U = \xi$ in (36), we get

$$[\lambda^2 + \epsilon\lambda(n-1) - 2(n-1)^2\epsilon^2]\eta(X) = 0. \quad (37)$$

In this case , since $\eta(X) \neq 0$, the relation (37) gives that

$$[\lambda^2 + \epsilon(n-1)\lambda - 2(n-1)^2\epsilon^2] = 0. \quad (38)$$

From the above equation it follows that the endomorphism Q has two different non-zero eigen val- ues,namely, $2(n-1)\epsilon$ and $-3(n-1)\epsilon$. Hence, we state the following theorem

Theorem 4.1. *Let M be an n-dimensional M -Projectively ϵ-kenmotsu manifold satisfying the con- dition* $R(X, Y).S = 0$, *then symmetric endomorphism Q of the tangent space corresponding to S has two diflerent non-zero eigen val- ues.*

5. Conservative M -Projective curvature tensor on ϵ-kenmotsu manifold

Definition 5.1. *An ϵ-kenmotsu manifold* (M, g) *is said to be M-Projective conservative if*

$$divM = 0, \quad (39)$$

where div denotes the divergence.

Taking the covariant derivative of (1), we get

$$(\nabla_U M)(X, Y)Z = (\nabla_U R)(X, Y)Z - \frac{1}{2(n-1)}[(\nabla_U S)(Y, Z)X - (\nabla_U S)(X, Z)Y + g(Y, Z)(\nabla_U Q)X - g(X, Z)(\nabla_U Q)Y] \quad (40)$$

Contracting with respect to U in (40), we obtain

$$(divM)(X, Y)Z = (divR)(X, Y)Z - \frac{1}{2(n-1)}[(\nabla_X S)(Y, Z) - (\nabla_Y S)(X, Z) + g(Y, Z)divQX - g(X, Z)divQY] \quad (41)$$

We know that

$$(divQ)(X) = \frac{1}{2}\nabla_X r. \quad (42)$$

$$(divM)(X,Y)Z = (divR)(X,Y)Z - \frac{1}{2(n-1)}[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) + \frac{1}{2}g(Y,Z)\nabla_X r - \frac{1}{2}g(X,Z)\nabla_Y r] \quad (43)$$

But from [7],we have

$$divR = (\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z). \quad (44)$$

Again, by virtue of (39) and (44) in (43) ,it reduces to

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = \frac{1}{2(2n-3)}[g(Y,Z)\nabla_X r - g(X,Z)\nabla_Y r]. \quad (45)$$

Putting $X = \xi$ in (45),we get

$$(\nabla_\xi S)(Y,Z) - (\nabla_Y S)(\xi,Z) = \frac{1}{2(2n-3)}[g(Y,Z)\nabla_\xi r - g(\xi,Z)\nabla_Y r]. \quad (46)$$

Further, we know that

$$(\nabla_\xi S)(X,Y) = \xi S(X,Y) - S(\nabla_\xi X)Y - S(X,\nabla_\xi Y) \quad (47)$$

$$(L_X g)(Y,Z) = L_X g(Y,Z) - g(L_X Y,Z) - g(Y,L_X Z) \quad (48)$$

Now put $X = \xi$ in (48) and using (10)

$$(L_\xi g)(Y,Z) = g(\nabla_Y \xi, Z) + g(Y,\nabla_Z \xi) \quad (49)$$

$$(L_\xi g)(Y,Z) = 2\epsilon[g(Y,Z) - \eta(Y)\eta(Z)] \quad (50)$$

Notice that $g(QX,Y) = S(X,Y)$ and using (50),we get

$$(L_\xi S)(Y,Z) = 2\epsilon[S(Y,Z) + (n-1)\eta(Y)\eta(Z)] \quad (51)$$

Making use of (10) and (51) in (47) ,we get

$$(\nabla_\xi S)(Y,Z) = 0, \quad (52)$$

which yields

$$\nabla_\xi r = 0. \quad (53)$$

In view of (45) and making use of (3), (10), (16) ,(52) and (53),we obtain

$$\epsilon S(Y,Z) + (n-1)g(Y,Z) = -\frac{1}{2(2n-3)}\eta(Z)dr(Y) \quad (54)$$

Now interchanging Y by ϕY and Z by ϕZ in (54) and using (3), (7) and (10), we get

$$S(Y,Z) = -\frac{1}{\epsilon}(n-1)g(Y,Z) + (\epsilon-1)\eta(Y)\eta(Z) \quad (55)$$

Hence ,we state the following:

Theorem 5.1. *Let M be an n-dimensional M -Projective curvature tensor on ϵ-kenmotsu manifold is conservative, then M is an η-Einstein manifold and Ricci tensor of M has the form* $S(Y,Z) = -\frac{1}{\epsilon}(n-1)g(Y,Z) + (\epsilon-1)\eta(Y)\eta(Z)$

Theorem 5.2. *Let M be an n-dimensional M -Projective curvature tensor on ϵ-kenmotsu manifold is conservative, then M is an Einstein manifold if taking $\epsilon = 1$ and Ricci tensor of M has the form* $S(Y,Z) = -^{1}(n-1)g(Y,Z)$

6. Irrotational M -Projective curvature tensor on ϵ-kenmotsu manifold

Definition 6.1. *The rotation (curl) of M-Projective curvature tensor on an ϵ-kenmotsu manifold M is defined as,*

$RotM = (\nabla_U M)(X, Y)Z + (\nabla_X M)(U, Y)Z + (\nabla_Y M)(X, U)Z - (\nabla_Z M)(X, Y)U.$ (56)

In consequence of Binachi second identity for Riemannian connection ∇,(56) becomes

$RotM = -(\nabla_Z M)(X, Y)U.$ (57)

If the M-Projective curvature tensor is irrotatinal, then $curlM = 0$ and so by (57), we get

$(\nabla_Z M)(X, Y)U = 0,$ (58)

which gives

$\nabla_Z(M(X, Y)U) = M(\nabla_Z X, Y)U + M(X, \nabla_Z Y)U + M(X, Y)\nabla_Z U.$ (59)

Putting $U = \xi$ in (59),we obtain

$\nabla_Z(M(X, Y)\xi) = M(\nabla_Z X, Y)\xi + M(X, \nabla_Z Y)\xi + M(X, Y)\nabla_Z \xi.$ (60)

Now ,substituting $Z = \xi$ in (1) and using the relation (2, 3, 12, 14) and (18), we obtain

$M(X, Y)\xi = \lambda[\eta(X)Y - \eta(YX],$ (61)

where,

$$\lambda = \frac{1}{2} + \frac{\epsilon\lambda \cdot 1}{2(n-1)} \quad (62)$$

By virtue of (62) and (10) in (60),we have

$$M(X, Y)Z = \frac{\lambda}{\epsilon}[g(Z, X)Y - g(Z, Y)X]. \quad (63)$$

In view of (1) and (63), we have

$$\frac{\lambda}{\epsilon}[g(Z, X)Y - g(Z, Y)X] = R(X, Y)Z - \frac{1}{2(n-1)}[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] \quad (64)$$

Contracting above equation (64) over X and using (62),we get

$$S(Y, Z)\left(\frac{n}{n-1}\right) = g(Y, Z)\left[\frac{r}{2(n-1)} - \frac{\lambda}{\epsilon}(n-1)\right] \quad (65)$$

from (65) ,we have

$$r = -\frac{2\lambda}{\epsilon}(n-1)^2. \quad (66)$$

Thus, we state the following theorem:

Theorem 6.1. *If the M-Projective curvature tensor on an ϵ-kenmotsu manifold M is irrota- tional,then the manifold is an Einstein manifold with constant scalar curvature* $-\frac{2\lambda}{\epsilon}(n-1)^2$.

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