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Applications of Matrices in Modern Scenario

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Abstract

In the modern world, matrices are considered to be of extreme use as they can be applied in fields like Chemistry, Physics, Economics, Construction, Finance, etc. In this work, the key applications empowered by the framework hypothesis in two significant fields of interest in graphic designing, in particular video game designing. The paper centers on how matrices and their operations play a significant role in designing and manipulating the structure on a screen to make it realistic. Among the significant applications in video games, it has a significant role in Adobe Photoshop how we put our structures in suggested dimensions and move according to our comfort in it and this is possible because of matrices. The study brings up the significant commitment made by cryptography, and the security process it carries with it so that no other person in between can decode the message and the message is delivered safely to the intended person. The significance pretended by grids in addressing and handling advanced pictures is portrayed by a few illustrative applications. This paper covers many other applications in matrices of how it is used in chemistry, physics, medical science, geology, economics, and many more.

Keywords

Matrix theory, Rectangular array, linear equation, graphic designing, cryptography

1. Introduction

Truly, it was not the matrix but rather a specific number related with a square exhibit of numbers considered the determinant that was first perceived. Just steadily did the possibility of the matrix as a logarithmic element arises. The term matrix was presented by the nineteenth century English mathematician James Sylvester, however it was his companion the mathematician Arthur Cayley who fostered the logarithmic part of lattices in two papers during the 1850s. Cayley previously applied them to the investigation of frameworks of linear equations, where they are still extremely valuable [1]. The commencement of matrix mechanics by Heisenberg, Born and Jordan prompted considering matrices with limitlessly numerous lines and columns. Later, von Neumann completed the numerical plan of quantum mechanics, by additional creating utilitarian scientific ideas, for example, straight administrators on Hilbert spaces, which, generally talking, compare to Euclidean space, however with a limitlessness of autonomous bearings.

2. Matrices

A rectangular arrangement of numbers in form of horizontal and vertical lines is called a matrix. The horizontal lines are called rows and vertical lines are called columns [2].

It is represented in form of:

$$\begin{pmatrix} i_{11} & \cdots & i_{1*n} \\ \vdots & \ddots & \vdots \\ i_{m*1} & \cdots & i_{m*n} \end{pmatrix}$$

In the above form, m*n is order of matrix. Therefore, in a matrix there are mn elements. Some basic types of matrices for example are,

- $\binom{3}{7}$, a column matrix of order 2*1
- $\begin{pmatrix} 2 & 5 \end{pmatrix}$, a row matrix of order 1*2
- $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, an identity matrix of order 2*2
- $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, a null matrix of order 2*2
- $\begin{pmatrix} 3 & 2 \\ 7 & 7 \end{pmatrix}$, a square matrix of order 2*2
- $\begin{pmatrix} 3 & 4 & 7 \\ 7 & 6 & 10 \end{pmatrix}$, a rectangular matrix of order 2*3

3. Types of Matrices

a) Transpose matrix [3]

A matrix is said to be transpose when rows of a matrix is represented as column for another matrix. Its notation is M^t. Suppose a matrix,

$$M = \begin{pmatrix} 2 & 4 \\ 7 & 8 \end{pmatrix}$$

$$M^t = \begin{pmatrix} 2 & 7 \\ 4 & 8 \end{pmatrix}$$

b) Symmetric matrix

When transpose matrix is equal to the original matrix, it is known as symmetric matrix.

$$P = P^t \quad \begin{pmatrix} p & q & r \\ q & t & z \\ r & z & y \end{pmatrix}$$

c) Skew- symmetric matrix

When transpose is equal to the negative of a matrix, it is called skew- symmetric matrix.

$$P = -P^t$$

$$\begin{pmatrix} 0 & n & i \\ -n & 0 & s \\ -i & -s & 0 \end{pmatrix} is\ a\ skew - symmetric\ matrix.$$

d) Inverse matrix

A matrix is inverse if product of two matrices gives identity matrix, following the property of cumulativeness for multiplication i.e. PQ=QP=I

For example,

$$P = \begin{pmatrix} 7 & 2 \\ 4 & 1 \end{pmatrix} \; and \; Q = \begin{pmatrix} 1 & -2 \\ -4 & 7 \end{pmatrix}$$

$$PQ = \begin{pmatrix} 7 & 2 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -4 & 7 \end{pmatrix}$$

$$= -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -I$$

$$QP = \begin{pmatrix} 1 & -2 \\ -4 & 7 \end{pmatrix}\begin{pmatrix} 7 & 2 \\ 4 & 1 \end{pmatrix}$$

$$= -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -I$$

Thus, we can say P is inverse of Q or Q is inverse of P.

e) Hermitian matrix [4]

A matrix is said to be Hermitian when a matrix P conjugated and then the transpose of it results to original matrix P.

$$A = \overline{(A)}^t$$

For example,

$$P = \begin{pmatrix} 2 & 1-4i \\ 1+4i & 3 \end{pmatrix}$$

$$\bar{P} = \begin{pmatrix} 2 & 1+4i \\ 1-4i & 3 \end{pmatrix}$$

$$(\bar{P})^t = \begin{pmatrix} 2 & 1-4i \\ 1+4i & 3 \end{pmatrix} = P$$

f) Skew- Hermitian matrix

When a matrix A is equal to the negative conjugate transpose matrix, it is known as Skew- Hermitian matrix.

$$-A = \overline{(A)}^t$$

For example,

$$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$(\bar{A})^t = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = A$$

g) Orthogonal matrix

When product of two matrices, one being an original matrix P other being the transpose of it results in identity matrix. The matrix P is known as orthogonal matrix.

$$PP^t = I$$

For example,

$$P = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}, P^t = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$
$$PP^t = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

In this case we can say the inverse of original matrix should be the transpose.

h) Unitary matrix

A matrix is said to be unitary, when product of a matrix and conjugate transpose matrix results in identity matrix [5].

$$P(\bar{P})^t = I$$

For example,

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}}i \end{pmatrix}, (\bar{P})^t = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i \end{pmatrix}$$
$$P(\bar{P})^t = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}}i \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

We say that the inverse of matrix is conjugate transpose matrix.

4. Elementary operation on Matrices

Elementary operations or elementary transformations is carried out in two ways:-

- Elementary row operation
- Elementary column operation

The following are major operations carried:-

- Interchanging of rows or columns.
- Multiplication of elements of any row or column by a non-zero real number.
- Addition to the elements if any row or column, the corresponding elements of any other row or column multiplied by any non-zero real number.

Taking up an example, let a matrix

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 2 & 3 & -3 \end{pmatrix}$$

$$we\ obtain, \qquad B = \begin{pmatrix} 2 & 3 & -3 \\ 0 & 1 & -2 \end{pmatrix}, C = \begin{pmatrix} -2 & 1 & 0 \\ -3 & 3 & 2 \end{pmatrix},$$

$$D = \begin{pmatrix} 0 & 2 & -2 \\ 2 & 6 & -3 \end{pmatrix}, E = \begin{pmatrix} 0 & -1 & -2 \\ 2 & 0 & -3 \end{pmatrix}$$

- In matrix A, we interchange elements of first and second rows to obtain matrix B.
- In matrix A, we interchange elements of first and third column to obtain matrix C.
- In matrix A, we multiply the second column with a non-zero real number say 2 to obtain matrix D.
- In matrix A, we add the third column in the second column to obtain matrix E, in this operation the value of the third column does not change [6].

We can also apply other operations such as subtraction, division, etc., to the elements of the matrix either the row elements or column elements. Using elementary operations, we can even find the inverse by reducing a matrix say, P to identity matrix.

$$PP^{-1} = I$$

$$P = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

We apply elementary column operation to obtain the inverse,

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_2 \to C_2 + C_3$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$C_1 \to C_1 + 2C_2$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

$$interchange\ C_1 \leftrightarrow C_3$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix}$$

$$C_3 \to C_3 - C_1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$multiply\ C_2\ by - 1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

The inverse of P is,

$$P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

5. Application of Matrices

Matrices acts as foundation in Mathematics, Science, Economics and many other fields. Not only in theoretical aspect but matrices have real life applications [7], [8], [9], [10].

- Matrix theory has high applications in education, this concept is taught in higher schools as a topic in the subject of mathematics, as to how we solve matrix problems associated with different situations, basically operations like addition, subtraction, multiplication.
- In physics, matrices help in finding the position, velocity of an electron in the orbit so that we can predict where the electron is. Kirchoff's law of voltage and current, optics and electrical circuits uses matrices.
- In medical field, matrices are used in CAT scans, MRIs. Matrices integrates and unify on several researches to find and establish data in a more precise and efficient manner, even for the patients to understand in some cases.
- Whenever we have to solve matrices of higher order we use MATLAB programming to find values like determinant, rank, eigen values.
- In Adobe Photoshop matrices are used to edit images by presenting the pixels of the image in number form.
- Artificial intelligence uses matrices for robot movements which are programmed by calculating matrices.
- In google, matrices are used for ranking the web pages using algorithms.
- In architecture, matrices play an important role by using coordinate points to place construction buildings in accurate dimensions.
- In geology, matrices are helpful for recording seismic surveys.
- Matrices are used by scientists to represent data of experiments.
- Matrices are best examined for plotting regular surveys.

6. Matrices in Video Games

Video games are products of entertainment for young generation. These video games comprises of a lots of programming and creativity. It also involves computer graphics which are accomplished by a mathematical tool called matrices to construct and engineer a realistic animation [11], [12], [13], [14]. Video games use three main principles of linear transformation of matrices: translation, rotation, scaling to program 3-D games. It is very interesting if you represent a coordinate system in the form of matrix. Suppose point (3, 2) is represented as a column matrix $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Say there is a transforming matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ we obtain a transformed matrix.

Transformed matrix = Transforming matrix * Original column matrix

This is the general representation for transforming points.

a) Translation

This tool of linear transformation helps in moving an object from one position to another, as in video games the human figure, car-shaped polygon move from one position to another with the help of translation. This operation carried by translation from coordinate (x, y, z) to (x+dx, y+dy, z+dz).

Representing it in matrix form:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

dx, dy, dz are the distances travelled by coordinate (x, y, z) to reach coordinate (x', y', z').

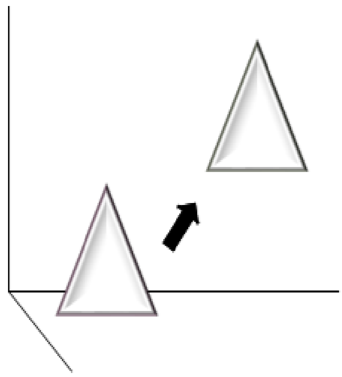

Figure. 1(A). 3-D graph showing how poly gon moves in video games

Figure. 1(B). Screen shot of a video game in which how a human polygon moves on a bike from one place to another without changing size and shape of any polygon.

b) Rotations

This tool of linear transformation is used to rotate polygon figures with respect to axis for 3-D formations.

i. Rotation about *z-axis*:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} xcos\theta - ysin\theta \\ xsin\theta + ycos\theta \\ z \end{pmatrix}$$

ii. Rotation about *x-axis*:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} x \\ ycos\theta - zsin\theta \\ ysin\theta + zcos\theta \end{pmatrix}$$

iii. Rotation about *y-axis*:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} zsin\theta + xcos\theta \\ y \\ zcos\theta - xsin\theta \end{pmatrix}$$

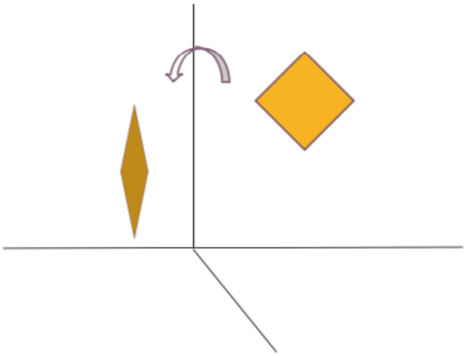

Figure 2(a). Rotation of a dimond polygon about X axis

Figure 2(b). In FIFA 20 a footboll as polygon rotates w.r.to axis

c) Scaling

This tool of linear transformation helps in changing the size either by enlarging or reducing the polygon figure. This operation carried by scaling for transforming coordinates from (x, y, z) to (x.Sx, y.Sy, z.Sz). [15], [16], [17]

Representing it in matrix form:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & Sz \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x.Sx \\ y.Sy \\ z.Sz \end{pmatrix}$$

In video games, to make it realistic we use this tool to magnify or reduce polygon figures like a box as required in specific games.

When we want to scale the cylinder polygon, we use scaling matrix at each point to enlarge the size of polygon.

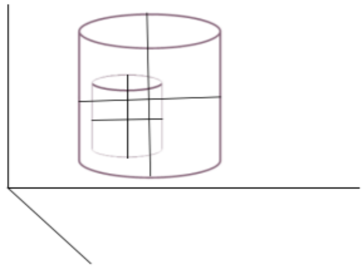

Figure 3. Scaling of Cylender Polygon

d) Reflection

This tool of linear transformation helps in viewing the reflection of the polygon figures like a tree, mountain, or a human figure sometimes for making it realistic as wherever there is a mirror or a sea formation, we reflect the respective polygon figures in different video games. In this we have to select between planes xy, yz, zx.

i. Reflection through xy plane:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ -z \end{pmatrix}$$

ii. Reflection through yz plane:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x \\ y \\ z \end{pmatrix}$$

iii. Reflection through zx plane:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ -y \\ z \end{pmatrix}$$

Example

If we want to reflect the moon through xz plane, where moon has coordinates as (4,2,3), (1,2,3), (2,2,1), (2,1,0), (0,0,0), (1,0,0) we use the reflected matrix

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 & 2 & 2 & 0 & 1 \\ 2 & 2 & 2 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$reflected\ matrix = \begin{pmatrix} 4 & 1 & 2 & 2 & 0 & 1 \\ -2 & -2 & -2 & -1 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Below graph shows the reflection of moon polygon.

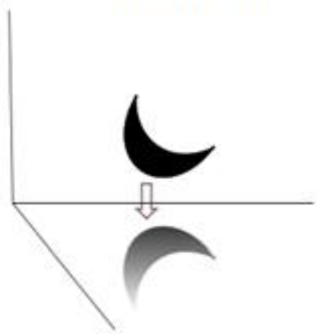

Figure 4. The reflection property is necessary in video games whenever we are striving for realistic effect whenever polygon pass through a surface which supports reflection.

e) Shearing

This tool of linear transformation helps in distorting the shape of polygon figures, assuming a car while moving in a video game when hit by another polygon in front of it, that part of the car gets rendered. This operation is carried about respective axis [18], [19], [20].

i. *z-axis* shearing:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & shx \\ 0 & 1 & shy \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
$$= \begin{pmatrix} x + z.shx \\ y + z.shy \\ z \end{pmatrix}$$

ii. *x-axis* shearing:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ shy & 1 & 0 \\ shz & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
$$= \begin{pmatrix} x \\ y + x.shy \\ z + x.shz \end{pmatrix}$$

iii. *y-axis* shearing:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & shx & 0 \\ 0 & 1 & 0 \\ 0 & shz & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
$$= \begin{pmatrix} x + y.shx \\ y \\ z + y.shz \end{pmatrix}$$

shx, shy, shz are the distortions occurred in x, y, z coordinates. Supposedly, in z.shy we are shearing y coordinate towards z and this goes with other shearing values.

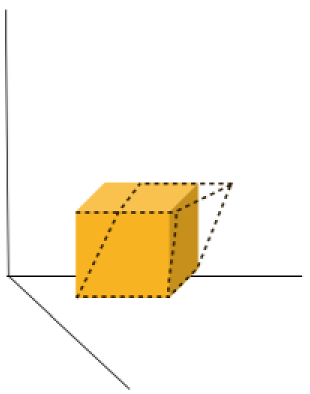

Figure 5(a). 3-D graph is constructed which shows x-axis shearing using shearing matrix.

Figure 5(b). Car Polygon has ben distorted as it mighnt have met with some accident, the car polygon follows the shearing matrix rule for it.

f) Projection

This tool of linear transformation helps in viewing by transforming polygon figures from 3-D to 2-D on the screen.

i. Parallel Projection

In parallel projection, we ignore the z-coordinate to transform the dimension from 3-D to 2-D. The parallel projection line transforming the dimensions is called projection vector. The size of the polygon figure doesn't change passing through the projection vector. Representing it in matrix form:

$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$$

ii. Perspective Projection

Video game designers try to make these games as real as possible for example, if a polygon figure is close it seems to be bigger and vice versa. So to carry out that vision, designers use perspective projection where parallelism is not applicable. We generally see points of edges converge at one point known as vanishing point, like a rail-track seems to converge till that point where our eyes can see, thus same is carried in video games for realistic effect [21]. Representing it in matrix form:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ \frac{z}{d} \end{pmatrix}$$

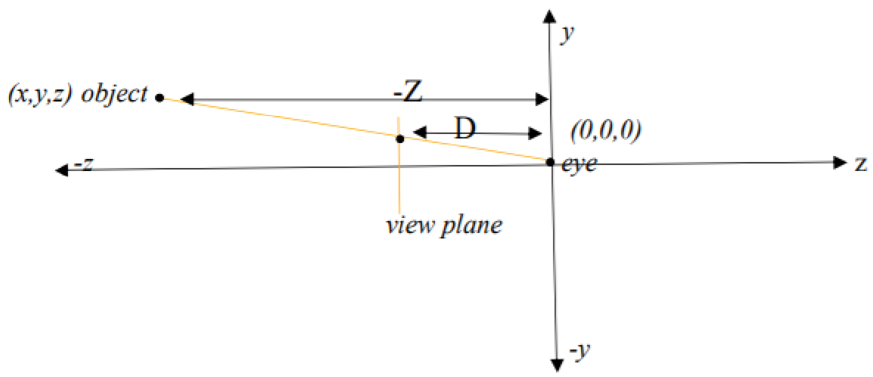

Figure 6(a). Perspective projection, D- distance of eye to plane, Z-distance of eye to plane.

Figure 6(b). View for the player through scope of a sniper.

g) Matrices in Cryptography

Cryptography is a technique for securing information and communicate via codes. The codes guarantee that only the intended receivers can read and process the messages. "Crypt" means hidden and "graphy" means writing. Mathematical tools help in deriving concepts for cryptography. It uses algorithms or rule-based calculations to send messages in ways that are hard to read. These algorithms create keys to control signing and verification so as to protect data privacy and its confidentiality. Cryptography is mainly used for sending e-mails, bank account numbers and transactions as well [22].

Suppose a conversation between two beings on WhatsApp is taking place, B1 is sending a message "BEHIND THE SCENES" to B2. This is saved in WhatsApp server as "BEHIND THE SCENES" so as to avoid any errors, it should be encrypted so that no one can understand. This process is known as encryption.

So, to follow the process, alphabets are converted to numeric values.

A	B	C	. . . .	X	Y	Z
1	2	3	. . . .	24	25	26

Now we encrypt the above message using the encryption or coding matrix then $\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix}$ we decode the received message using the inverse of the encrypted matrix.

BEHIND THE SCENES

≅(B E H)(I N D)(0 T H)(E 0 S)(C E N)

= (2 5 8) (9 14 4) (0 20 8) (5 0 19) (3 5 14) (5 19 0)

We apply matrix multiplication to un-coded row matrix with encoding matrix to obtain coded row matrix.

$$(2 \quad 5 \quad 8)\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix} = (20 \quad -7 \quad 2)$$

$$(9 \quad 14 \quad 4)\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix} = (41 \quad -23 \quad 9)$$

$$(0 \quad 20 \quad 8)\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix} = (48 \quad -20 \quad 0)$$

$$(5 \quad 0 \quad 19)\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix} = (24 \quad -5 \quad 5)$$

$$(3 \quad 5 \quad 14)\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix} = (27 \quad -8 \quad 3)$$

$$(5 \quad 19 \quad 0)\begin{pmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{pmatrix} = (43 \quad -24 \quad 5)$$

Coded message is:

$(20 \quad -7 \quad 2)\ (41 \quad -23 \quad 9)\ (48 \quad -20 \quad 0)\ (24 \quad -5 \quad 5)\ (27 \quad -8 \quad 3)\ (43 \quad -24 \quad 5)$

Now to decode the message we find the decoding matrix by finding out the inverse of encrypted matrix by using,

$$A^{-1} = \frac{1}{|A|} adj\ A$$

$$A^{-1} = \begin{pmatrix}0 & 0 & 1\\ 0 & -1 & 2\\ 1 & -1 & 1\end{pmatrix}$$

We decode the message by post-multiplying by A^(-1)

$$(20 \quad -7 \quad 2)\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix} = (2 \quad 5 \quad 8)$$

$$(41 \quad -23 \quad 9)\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix} = (9 \quad 14 \quad 4)$$

$$(48 \quad -20 \quad 0)\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix} = (0 \quad 20 \quad 8)$$

$$(24 \quad -5 \quad 5)\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix} = (5 \quad 0 \quad 19)$$

$$(27 \quad -8 \quad 3)\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix} = (3 \quad 5 \quad 14)$$

$$(43 \quad -24 \quad 5)\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix} = (5 \quad 19 \quad 0)$$

Decoded message is:

$(2 \quad 5 \quad 8)\ (9 \quad 14 \quad 4)\ (0 \quad 20 \quad 8)\ (5 \quad 0 \quad 19)\ (3 \quad 5 \quad 14)\ (5 \quad 19 \quad 0)$

Hence, required message delivered is

(B E H) (I N D) (0 T H) (E 0 S) (C E N)≅BEHIND THE SCENES

In this way we send messages safely without any occurrence of cyber-crime.

Conclusion

Matrices is an intriguing subject with regards to the field of linear algebra. Indeed, even in training it is considered as one of the least demanding subjects under mathematics. Utilizing matrices standard of linear transformations assume a significant part in present day situation like in computer game designs. In cryptography utilization of matrices is fundamental for getting information securely, utilizing basic rule of inverse matrices. Out and out, matrices are advantageous for additional turn of events.

References

[1] D. R. Camp and P. Y, "Secret codes with matrices," Mathematics Teacher, vol. 78, no. 9, pp. 676–680, 1985.

[2] A. Baker, Matrix Groups. London, England: Springer London, 2002.

[3] R. A. Beauregard and J. B. Fraleigh, A First Course In Linear Algebra: with Optional Introduction to Groups, Rings, and Fields. Boston: Houghton Mifflin, 1973.

[4] L. I. Schiff and J. Bandhyopadhyay, "Buy Quantum Mechanics book online at low prices in India," Amazon.in. [Online]. Available: https://www.amazon.in/Quantum-Mechanics-Leonard-I-Schiff/dp/1259062864. [Accessed: 06-Dec-2020].

[5] A. Cayley, "A memoir on the theory of matrices," Philosophical Transactions of the Royal Society of London, Volume 148, Pages 17-37.

[6] C. Arthur, The collected mathematical papers of Arthur Cayley. Charleston, SC: BiblioLife, 2009.

[7] H. Anton and C. Rorres, "Elementary linear algebra applications version." 1994.

[8] S. Abraham, Elementary cryptanalysis a, a mathematical approach. Mathematical Association of America, mathematical library, 1966.

[9] A. G. Konheim, Cryptography: A Primer, First Edition; Third Printing. Nashville, TN: John Wiley & Sons, 1981.

[10] T. S. Blyth and E. F. Robertson, Basic linear algebra, 2nd ed. Guildford, England: Springer, 2002.

[11] H. Anton, “Elementary Linear Algebra”, John Wiley & Sons, July 2019, ISBN: 978-1-119-62569-8.

[12] X. Huang and S. G. Krantz, “On a problem of Moser,” Duke Math. J, vol. 78, pp. 213–228, 1995.

[13] O. D. Kellog, “Harmonic functions and Green’s integral,” Trans. Amer. Math. Soc, vol. 13, pp. 109–132, 1912.

[14] K. M. Rasulov and B. F. Fatulaev, Differ. Equ., vol. 38, no. 2, pp. 274–278, 2002.

[15] “Doklady Mathematics,” Springer.com. [Online]. Available: https://www.springer.com/journal/11472. [Accessed: 06-Dec-2020].

[16] “Introduction to Linear Algebra, 5th Edition,” Mit.edu. [Online]. Available: https://math.mit.edu/~gs/linearalgebra/. [Accessed: 06-Dec-2020].